Progress in Aeronautical Sciences, Vol. 7, edited by Küchemann et al., Pergamon Press, New York, 1966.

[14] Squire, H. B., "Analysis of the Vortex-Breakdown Phenomenon," Part I, Rept. 102, 1960, Aeronautics Department, Imperial College, London.

[15] Benjamin, T. B., "Theory of the Vortex Breakdown Phenomenon," *Journal of Fluid Mechanics*, Vol. 14, Pt. 4, Dec. 1962, p. 593.

[16] Benjamin, T. B., "Significance of the Vortex Breakdown Phenomenon," *Transactions of the American Society of Mechanical Engineers, Journal of Basic Engineering*, Vol. 87, No. 2, 1965, pp. 518, 1091.

[17] Benjamin, T. B., "Some Developments in the Theory of Vortex Breakdown," *Journal of Fluid Mechanics*, Vol. 28, Pt. 1, April 1967, p. 65.

[18] Ludwieg, H., "Zur Erklärung der Instabilitat der über angestellten Deltaflügeln auftreten freien Wirkbelkerne," *Zeitschrift für Flugwissenschaften*, Vol. 10, 1962, p. 242.

[19] Ludwieg, H., "Experimentelle Nachprüfung der Stabilitätstheorien für reibungsfreie Strömungen mit schraubenformigen Strömlinien," *Zeitschrift für Flugwissenschaften*, Vol. 12, 1964, p. 304.

[20] Jones, J. P., "The Breakdown of Vortices in Separated Flow," Rept. 140, 1960, University of Southampton Aeronautics and Astronautics Department.

[21] Hall, M. G., "On the Occurrence and Identification of Vortex Breakdown," TN 66283, 1966, Royal Aircraft Establishment, Farnborough Hants.

[22] Bossel, H. H., "Vortex Breakdown Flowfield," *The Physics of Fluids*, Vol. 12, No. 3, 1969, p. 498.

[23] Leibovich, S., "Weakly Non-Linear Waves in Rotating Fluids," *Journal of Fluid Mechanics*, Vol. 42, Pt. 4, Dec. 1970, p. 803.

[24] Pedley, T. J., "On the Instability of Viscous Flow in a Rapidly Rotating Pipe," *Journal of Fluid Mechanics*, Vol. 35, Pt. 1, Jan. 1969, p. 97.

[25] Talbot, L., "Laminar Swirling Pipe Flow," *Journal of Applied Mechanics*, Vol. 21, No. 1, March 1954, p. 1.

[26] Joseph, D. D. and Munson, B. R., "Global Stability of Spiral Flow," *Journal of Fluid Mechanics*, Vol. 43, Pt. 3, Sept. 1970, p. 545.

[27] Henderson, F. M., *Open Channel Flow*, Macmillan, New York, 1966, pp. 45, 215–218.

SEPTEMBER 1971 AIAA JOURNAL VOL. 9, NO. 9

Compressible Turbulent Boundary-Layer Heat Transfer to Rough Surfaces

D. E. Nestler*
General Electric Company, Philadelphia, Pa.

The semiempirical correlation of Owen and Thomson for heat transfer to rough surfaces in subsonic turbulent flow is extended to supersonic conditions. The correlation relates heat flux to skin friction, which is determined by Goddard's method for rough surfaces, modified empirically to include nonadiabatic wall conditions. The method is verified by comparison with existing experimental results for Mach numbers from 3 to 4.9. Application to a typical re-entry vehicle ablated nose shape indicates that the heat flux increases rapidly, then reaches a maximum, as roughness increases. The example indicates that Reynolds analogy is invalid for rough surfaces.

Nomenclature

B = sublayer Stanton number
C_f = local skin-friction coefficient
K = equivalent sand grain roughness height
M = Mach number
Pr = Prandtl number
R_N = nose radius
$Re_K{}^*$ = roughness Reynolds number (Eq. 8)
Re_s = Reynolds number based on local properties and wetted length; $Re_s = \rho_e U_e S/\mu_e$
St = Stanton number
S = wetted length
T = temperature
U_τ = shear velocity; $U_\tau = (\tau_w/\rho_w)^{0.5}$
U_∞ = freestream velocity
α = empirical constant (Eq. 2)
θ = central angle; $\theta = S/R_N$
θ_N = nose semivertex angle
ρ = density
μ = viscosity

Subscripts

aw = adiabatic wall
e = edge of boundary layer
i = incompressible
O = smooth surface
R = recovery
W = wall

Presented as Paper 70-742 at the AIAA 3rd Fluid and Plasma Dynamics Conference, Los Angeles, Calif., Jun 29–July 1, 1970; submitted July 17, 1970; revision received December 2, 1971.

* Consultant, Aerothermodynamics; Re-entry and Environmental Systems Division. Member AIAA.

Introduction

SURFACE roughness can cause significant increases in convective heat flux and skin friction. Recent developments in re-entry vehicle technology have indicated the importance of roughness in two regions: 1) ablative noses, and 2) ablative frustums having cross-hatching patterns. Materials such as graphite or ablative composites consisting of filler and binder materials can develop surface roughness of from 1 to 10 mils, which can be of the same order as the boundary-layer thickness on the nose during ballistic re-entry. This magnitude of roughness is sufficient to trip the laminar boundary layer to turbulent flow, as illustrated by the wind-tunnel data of Deveikis and Walker,[1] shown in Fig. 1. The increase in turbulent heat flux due to roughness is significant in determining the amount of ablation and shape change of the nose, as discussed by Welsh.[2] The cross-hatching ablation pattern phenomenon is common to the three major generic classes of heat shield materials (sublimers, melters, and charring ablators), and occurs in supersonic turbulent flow, as discussed by Laganelli and Nestler.[3] Much larger roughness amplitude of the wavy wall type can develop in cross-hatching ablation than in stagnation region

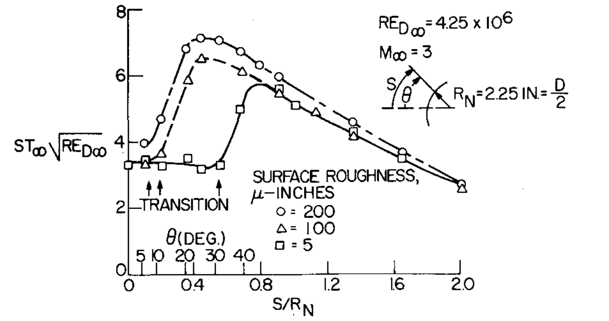

Fig. 1 Effect of surface roughness on transition location.[1]

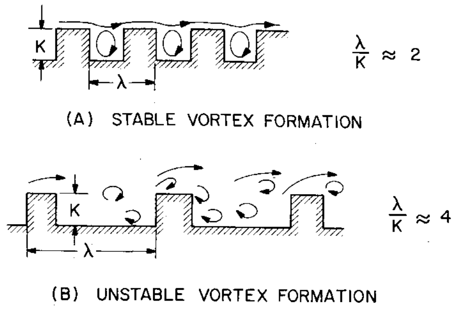

Fig. 3 Types of flow for two-dimensional roughness (Perry, et al[16]).

ablation. However, premature transition due to roughness can lead to ablated shapes having supersonic turbulent flow on the nose tip itself, such as the test model shown in Fig. 2. The large scale roughness on this nose appears to be the remains of a mature cross-hatching pattern which has become partially obliterated, similar to results discussed by Larson and Mateer.[4] A large portion of the increased nose ablation rates at high pressures appears to be caused by surface roughness effects on boundary-layer transition and increased heating, rather than solely by mechanical erosion as previously inferred by Kratsch et al.[5]

Semiempirical correlations of heat transfer to rough surfaces have been developed for subsonic flow. No established method exists, however, for predicting heat flux to rough surfaces in turbulent, supersonic flow. The objective of this paper is to develop such a method.

Review of Previous Investigations

The literature is abundant with studies of various aspects of turbulent flow over roughened surfaces, including boundary-layer structure, wall shear, and heat flux. The vast majority of published works deal with incompressible flow through tubes and channels, including the classical sand grain experiments of Nikuradse,[6] the later studies of Cope[7] and Nunner,[8] and the more recent investigations of Liu et al,[9] Gowen and Smith,[10] Dipprey and Sabersky,[11] and Migay.[12] Studies of subsonic flow over plates having various types of two and three-dimensional roughness have been reported by Owen and Thomson,[13] Doenecke,[14] Betterman,[15] and Perry et al.[16]

Two very similar methods of correlating rough surface turbulent heat-transfer results were proposed independently by Dipprey and Sabersky[11] and Owen and Thomson.[13] In both methods, a sublayer Stanton number is introduced and related empirically to roughness Reynolds number and Prandtl number. The principal difference between the methods is that Dipprey and Sabersky's model treats the flow between roughness elements as a two-dimensional cavity flow slowed down by friction within a thin layer adjacent to the surface and separated from the main flow by a mixing region, while Owen and Thomson consider the flow in the sublayer as three-dimensional, with horseshoe eddies wrapping around individual roughness elements. Owen and Thomson's model appears to be more appropriate for application to roughness developed from ablation.

Correlations such as those of Owen and Thomson[13] or Dipprey and Sabersky[11] fail to allow for the effect of roughness shape or spacing. The spacing between roughness elements has been shown by Perry et al[16] and Liu et al[9] to affect the type of cavity flow which develops. Two distinct types of flow are shown in Fig. 3; for spacing-to-height ratios on the order of 2, relatively stable vortex formations exist within the cavities between roughness elements, with negligible eddy shedding from the elements into the flow, whereas for spacing-to-height ratios on the order of 4, unstable vortex formations and eddy-shedding occur.

The flow visualization studies of Townes and Sabersky[17] are of great potential significance in identifying the importance of relative roughness height to the formation of a steady vortex pattern. For roughness Reynolds numbers below a certain threshold value, unsteady cavity flow existed in square slots, with alternate formation and dispersion of vortex patterns. When the threshold value of roughness Reynolds number was exceeded, stable vortex flow developed. The threshold region was identified as that part of the "roughness transition" region just below the "fully rough" region. It was suggested that the resistance to heat transfer should reach a minimum at this point, which is equivalent to saying that a maximum value of heat flux will be obtained at some value of roughness Reynolds number. This same observation was made by Dipprey and Sabersky[11] from experimental heat transfer data.

Turning to the problem of compressible turbulent flow over rough surfaces, it is noted that the few available "theories" for heat transfer (e.g. VanDriest,[18] and Fenter[19]) assume the validity of Reynolds analogy. Since a growing body of ex-

Fig. 2 Ablated phenolic nylon nose tip, Malta rocket exhaust.

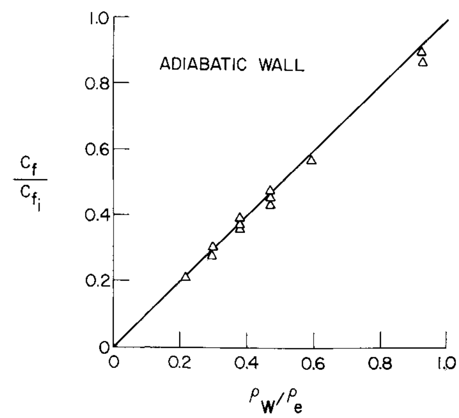

Fig. 4 Effect of compressibility on skin-friction coefficient ratio for a rough surface (Goddard[24]).

perimental data indicates that skin friction continues to increase as roughness height increases; in contrast to the maximum reached by heat flux, it appears that Reynolds analogy is not applicable for rough surfaces. Hence, it becomes necessary to develop a new approach for engineering predictions of rough surface heat flux in supersonic flow.

Experimental heat flux data on rough surfaces in supersonic turbulent flow include those of Fenter,[19] Jones,[20] Young[21] and Deveikis and Walker.[1] None of these studies involved extensive variation of more than one of the several potentially significant parameters, such as Mach number, Reynolds number, wall cooling ratio, and type of roughness. Hence the development of an empirical correlation of these data is quite risky, and must be regarded as preliminary pending further investigations.

Derivation of Increased Heat Flux Due to Roughness

The relation proposed by Owen and Thomson for turbulent boundary-layer flow over a rough flat plate may be written as

$$St^{-1} = (U_e/U_\tau)[(U_e/U_\tau) + B^{-1}] \tag{1}$$

in which B is the sublayer Stanton number. By empirical analysis of subsonic heat flux data for two-and-three-dimensional roughness, Owen and Thomson obtained the following expression for B:

$$B^{-1} = \alpha(\rho U_\tau K/\mu)^{0.45} Pr^{0.8} \tag{2}$$

in which α varied between 0.45 and 0.7, with a mean value of $\alpha = 0.52$. Although Eqs. (1) and (2) permit computation of heat flux to rough surfaces, their applicability to supersonic flow conditions is unknown.

For comparison with available experimental data for rough surfaces in supersonic flow, it is convenient to derive a relation for St/St_o, the ratio of Stanton number with roughness to Stanton number for a smooth surface. Noting that $B^{-1} \rightarrow 0$ as $K \rightarrow 0$, Eq. (1) yields for a smooth surface

$$St_o^{-1} = (U_e/U_{\tau_o})^2 \tag{3}$$

Dividing Eq. (3) by Eq. (1) yields

$$St/St_o = (U_\tau/U_{\tau_e})^2/[1 + (U_\tau/U_e)(1/B)] \tag{4}$$

From the definition of U_τ, it follows that

$$U_\tau/U_{\tau_e} = (C_f/C_{fo})^{0.5} \tag{5}$$

and also

$$U_{\tau_e}/U_e = [(C_{fo}/2)(\rho_e/\rho_w)]^{0.5} \tag{6}$$

Introducing Eqs. (2), (5), and (6) into Eq. (4) and simplifying leads to the final expression

$$St/St_o = (C_f/C_{fo})/[1 + \alpha(C_f/C_{fo})^{0.725} \times (U_{\tau_o}/U_e)(Re_K{}^*)^{0.45}(Pr)^{0.8}] \tag{7}$$

in which

$$Re_K{}^* = \rho_w U_{\tau_o} K/\mu_w \tag{8}$$

(assuming that wall values of ρ and μ are significant for compressible flow analysis). In Eq. (8), the shear velocity $U_{\tau o}$ is computed from the smooth surface value of wall shear.

Eq. (7) relates the increase in Stanton number due to roughness to the increase in skin-friction coefficient due to roughness and to three other dimensionless parameters: $U_{\tau o}/U_e$, $Re_K{}^*$, and Pr. Reynolds analogy does not hold for flow over rough surfaces, according to Eq. (7), since C_f will increase more rapidly than St as roughness increases.

The evaluation of St/St_o from Eq. (7) requires knowledge of C_f/C_{fo}, the increase in local skin-friction coefficient due to roughness. The effect of roughness on turbulent skin friction

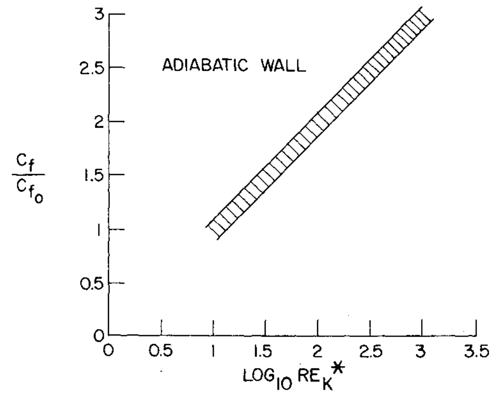

Fig. 5 Correlation of skin-friction coefficient for rough surfaces.

has received considerable investigation in low-speed flow, as summarized by Clauser.[22] Measurements for compressible flow conditions have been made by Wade,[23] Goddard,[24] Fenter,[19] and Young.[21] An extremely significant contribution was made by Goddard, who showed that the effect of compressibility over a Mach number range from 0.7 to 4.5 was simply a reduction in wall density as Mach number increased (Fig. 4). Goddard also showed that the skin friction increase due to roughness correlated with the roughness Reynolds number $\rho_w U_\tau K/\mu_w$, independent of Mach number. Goddard's results for C_f/C_{fo} have been replotted vs $\log_{10} Re_K{}^*$ in Fig. 5 to avoid iteration in determining C_f. The data of Fig. 5 were limited to adiabatic flow conditions; however, Young's results at Mach 4.9 were for cooled-wall conditions, covering a range of T_w/T_o from 0.5 to 0.9. Young found that as the wall temperature was reduced, the skin friction dropped below the Goddard value for adiabatic walls. Reasoning that the effect of heat transfer was to produce a density gradient near the wall such that the effective density at the roughness surface was less than the wall value, Young developed a reference temperature method that correlated his results (Fig. 6). The relation obtained by Young for C_f was

$$C_f/C_{fi} = 0.365\ (T_e/T_R) + 0.635\ (T_e/T_w) \tag{9}$$

in which C_{fi} is the incompressible value of C_f including roughness effect. As indicated in Fig. 6, Goddard's expression for adiabatic wall conditions is

$$(C_f/C_{fi})_{aw} = T_e/T_w \tag{10}$$

Eq. (9) should yield reasonable estimates of roughness effects on turbulent shear for subsonic or supersonic flow, including wall cooling effects. More data at intermediate supersonic

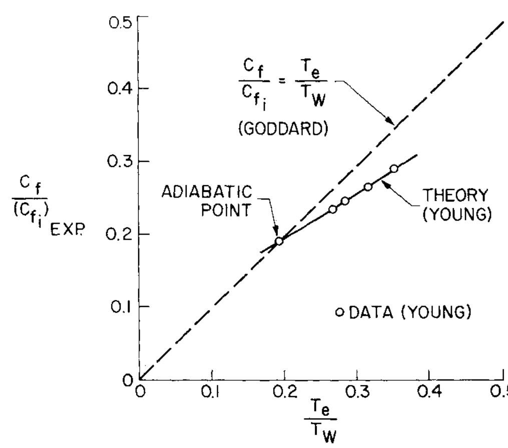

Fig. 6 Effect of wall temperature ratio on skin-friction coefficient ratio at $M_e = 4.93$ (Young[21]).

Table 1 Summary of test conditions and pertinent parameters for heat flux to rough surfaces in turbulent flow

Investigator	Model Geometry	Type of Roughness	K (in) $\times 10^{-3}$	M_e	T_W/T_e	T_W/T_R	$Re_x \times 10^6$	Re_K^*	C_f/C_{f_o} (See Note)	(St/St_o) Eq. 7	(St/St_o) Exp.	Max. Percent Error
Fenter[19]	Cone-Cylinder	V-Thread	1.1	3.04	2.1	0.79	8.0	8.0	1.2	1.15	1.10/1.18	5.0
			7.6	3.04	2.1	0.79	8.3	57.0	2.0	1.72	1.82/1.89	-10.0
Young[21]	Flat Plate	V-Grooves	5.0	4.93	2.8	0.525	5.0	15.0	1.13	1.06	1.11	-4.7
			15.0	4.93	2.8	0.525	5.0	47.0	1.38	1.22	1.20	1.6
			5.0	4.93	4.0	0.75	9.0	13.0	1.12	1.04	1.17	-12.5
		90°	15.0	4.93	4.0	0.75	9.0	46.0	1.41	1.23	1.25	-1.6
Jones[20]	Cone	Machined Surface	0.2	4.25	3.0	0.71	40.0	6.7	1.0	1.0	1.0	--
Deveikis and Walker[1]	Sphere-ellipsoid	Sand-blasted	0.1	1.28	1.0	0.77	1.5	2.9	1.0	1.0	1.2	-20.0
			0.2	1.28	1.0	0.77	1.5	5.8	1.0	1.0	1.4	-40.0

Note: Experimental values of C_f/C_{f_o} were used when available (i.e., Fenter[19] and Young[21])

Mach numbers are of course desirable to confirm the empirical constants in this equation.

The final relation for C_f/C_{fo} including wall cooling is obtained by correcting the adiabatic results of Fig. 5 by the following relation derived by dividing Eq. (9) by Eq. (10):

$$C_f/C_{faw} = 0.365\,(T_w/T_R) + 0.635 \qquad (11)$$

A close approximation to the mean of the data spread in Fig. 5 is given by

$$(C_f/C_{fo})_{aw} = \log_{10} Re_K^* \qquad (12)$$
$$(Re_K^* \geq 10)$$

It is now assumed that Eq. (12) can be corrected for non-adiabatic (wall cooling) conditions by multiplying by the rhs of Eq. (11), yielding

$$C_f/C_{fo} = [0.365\,(T_w/T_R) + 0.635] \log_{10} Re_K^* \qquad (13)$$

Eq. (13) is the expression recommended to determine the term C_f/C_{f0} which appears in Eq. (7) for St/St_0. When perfect gas conditions do not apply, as in high-enthalpy arc tests or actual re-entry, T_w/T_R should be replaced by the enthalpy ratio h_w/h_R.

Correlation of Compressible Flow Data

It is reasonable to assume that Eq. (7), although derived from an incompressible flow correlation, may yield agreement with compressible flow data. Eq. (7) is essentially a Reynolds analogy for rough surfaces; and the normal flat plate Reynolds analogy is unaffected by compressibility, at least for Mach numbers up to 5 (Cary[25]). If compressibility effects on skin friction of rough surfaces are properly accounted for by Eq. (13), compressibility effects on heat flux should likewise be approximated by Eq. (7).

To test this hypothesis, the available experimental data for compressible flow conditions were evaluated by means of Eq. (7), assuming $\alpha = 0.52$. These data and the principal computed and measured parameters are summarized in Table 1. In general, the difference between the prediction of Eq. (13) and the measured values of St/St_o is less than 10%, which is within typical measurement accuracy. The exception to this excellent agreement is the data of Deveikis and Walker which fall 20–40% above prediction. Since their data were obtained on a sphere-ellipsoid body, while the other data were taken on cylinders, flat plates, and cones, it appears that a favorable longitudinal pressure gradient causes increases in roughness heat flux relative to zero pressure gradient conditions. An alternate explanation is the possibility that the sand blasted surface used by Deveikis and Walker had a larger effective roughness than reported. The experiments of Schlichting[26] on a large number of rough surfaces of various arrangements of three-dimensional elements showed that the equivalent sand grain roughness of closely packed arrays could be as much as four times the actual protrusion height. However, evaluation of the Deveikis-Walker data with the assumption $K_{eff} = 4\,K$ accounts for only about half the error; hence the pressure gradient effect is probably a real effect. Additional data on roughened spheres are desirable to resolve this question, due to the practical application of ablating nose-tips.

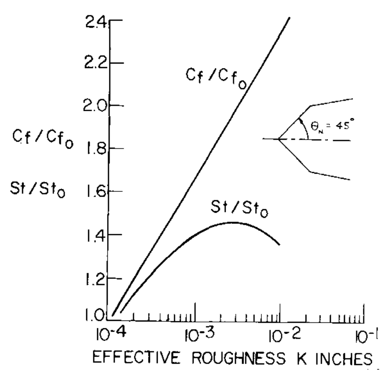

Fig. 7 Example of rough ablated nose tip (U_∞ = 20,000 fps, altitude = 40,000 ft, S = 1 in., T_w = 6,000 R, θ_N = 45°, Pr = 0.72).

Example of Application of Correlation

It is of interest to apply the correlation of Eq. (7) to an example of a nose-tip that has ablated to a biconic shape. The following conditions were assumed: U_∞ = 20,000 ft/sec, altitude = 40,000 ft, S = 1 in., T_W = 6000 R, θ_N = 45°, and Pr = 0.72. The smooth surface value of C_{fo} was computed to be .004 by the Eckert reference enthalpy method.†

† No allowance for reduction in C_{fo} due to ablative mass addition was made; if a large blowing rate exists, C_{fo} should be reduced for blocking effects as a first approximation to the combined effects of roughness and mass addition.

The resulting values of C_f/C_{f_o} and St/St_o are plotted vs roughness height in Fig. 7. Although skin friction continues to rise steadily as roughness increases, heat transfer is seen to reach a maximum at $K = .003$ in. The increase in C_f is seen to be much larger than the increase in St, indicating the invalidity of the usual Reynolds analogy even for approximation purposes. The maximum value of St/St_o is predicted to be 1.47, in contrast to the value of 3.0 assumed by Welsh[2] in his analysis of nose shape change.

Conclusions

The semiempirical correlation of Owen and Thomson for heat transfer to rough surfaces in subsonic turbulent flow has been used to derive a relation for the increase in heat flux due to roughness. This relation has been shown to yield adequate prediction of the available experimental heat-transfer data for rough surfaces in compressible flow conditions for a Mach number range of 3–4.9, for zero axial pressure gradient. Limited data on a blunt nose suggest that a favorable pressure gradient causes an increase in roughness heat flux relative to the zero pressure gradient value. Additional experimental data are needed to verify the validity of the present correlation over a wider range of Mach number, wall temperature ratio, and pressure gradient.

References

[1] Deveikis, W. D. and Walker, R. W., "Local Aerodynamic Heat Transfer and Boundary-Layer Transition on Roughened Sphere-Ellipsoid Bodies at Mach Number 3.0," TN D-907, Aug. 1961, NASA.

[2] Welsh, W. E., Jr., "Shape and Surface Roughness Effects on Turbulent Nose Tip Ablation," *AIAA Journal*, Vol. 8, No. 11, Nov. 1970, pp. 1983–1989.

[3] Laganelli, A. L. and Nestler, D. E., "Surface Ablation Patterns: A Phenomenology Study," *AIAA Journal*, Vol. 7, No. 7, July 1969, pp. 1319–1325.

[4] Larson, H. K. and Mateer, G. G., "Cross-Hatching—A Coupling of Gas Dynamics with the Ablation Process," AIAA Paper 68-670, Los Angeles, Calif., 1968.

[5] Kratsch, K. M., et al, "Graphite Ablation in High-Pressure Environments," AIAA Paper 68-1153, Williamsburg, Va., 1968.

[6] Nikuradse, J., "Laws for Flows in Rough Pipes," *VDI-Forschungsheft*, 361, Ser. B, Vol. 4, 1933; transl. TM 1292, 1950, NACA.

[7] Cope, W. F., "The Friction and Heat Transmission Coefficients of Rough Pipes," *Proceedings of the Institution of Mechanical Engineers*, Vol. 145, 1941, pp. 99–105.

[8] Nunner, W., "Heat Transfer and Pressure Drop in Rough Tubes," *VD1 Forschungsheft*, 455, Ser. B, Vol. 22, 1956, pp. 5–39; A.E.R.E. Lib/Transl. 786, 1958.

[9] Liu, C. K., Kline, S. J., and Johnston, J. P., "An Experimental Study of Turbulent Boundary Layer on Rough Walls," Rept. MD-15, (AD 647268), July 1966, Stanford University Dept. of Mechanical Engineering.

[10] Gowen, R. A. and Smith, J. W., "Turbulent Heat Transfer from Smooth and Rough Surfaces," *International Jounal of Heat and Mass Transfer*, Vol. 11, 1968, pp. 1657–1673.

[11] Dipprey, D. F. and Sabersky, R. H., "Heat and Momentum Transfer in Smooth and Rough Tubes at Various Prandtl Numbers," *International Journal of Heat and Mass Transfer*, Vol. 6, 1963, pp. 329–353.

[12] Migay, V. K., "Heat Transfer in Rough Pipes," *Heat Transfer-Soviet Research*, Vol. 1, No. 2, 1969, pp. 69–79.

[13] Owen, P. R. and Thomson, W. R., "Heat Transfer Across Rough Surfaces," *Journal of Fluid Mechanics*, Vol. 15, 1963, pp. 321–334.

[14] Doenecke, J., "Contribution a l'Etude de la Convection Forcee Turbulente le Long de Plaques Rugueuses," *International Journal of Heat and Mass Transfer*, Vol. 7, 1964, pp. 133–142.

[15] Betterman, D., "Contribution a l'Etude de la Convection Forcee Turbulente de Long de Plaques Rugueuses," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 153–164.

[16] Perry, A. E., Schofield, W. H., and Joubert, P. N., "Rough Wall Turbulent Boundary Layers," *Journal of Fluid Mechanics*, Vol. 37, 1969, pp. 383–413.

[17] Townes, H. W. and Sabersky, R. H., "Experiments on the Flow over a Rough Surface," *International Journal of Heat and Mass Transfer*, Vol. 9, 1966, pp. 729–738.

[18] VanDriest, E. R., "The Problem of Aerodynamic Heating," *Aeronautical Engineering Review*, Oct. 1956, pp. 26–41.

[19] Fenter, F. W., "The Turbulent Boundary Layer on Uniformly Rough Surfaces at Supersonic Speeds," Chance Vought Rept. RE-E9R-2, Dec. 1959.

[20] Jones, J. J., "Experimental Investigation of the Heat Transfer Rate to a Series of 20° Cones of Various Surface Finishes at a Mach Number of 4.95," Memo 6-10-59L, June 1959, NASA.

[21] Young, F. L., "Experimental Investigation of the Effects of Surface Roughness on Compressible Turbulent Boundary Layer Skin Friction and Heat Transfer" DRL-432, (AD-621085), May 1965, Univ. of Texas.

[22] Clauser, F. H., "Turbulent Boundary Layer," *Advances in Applied Mechanics*, Vol. IV, ed. by H. L. Dryden and T. von Kármán, Academic Press, New York, 1956, pp. 1–21.

[23] Wade, J. H. T., "An Experimental Investigation of the Effects of Surface Roughness on the Drag of a Cone-Cylinder Model at a Mach Number of 2.48," Univ. of Toronto, Institute of Aerophysics, Rept. 34, Sept. 1955.

[24] Goddard, F. E., Jr., "Effect of Uniformly Distributed Roughness on Turbulent Skin-Friction Drag at Supersonic Speeds," *Journal of the Aerospace Sciences*, Vol. 26, No. 1, Jan. 1959, pp. 1–15.

[25] Cary, A. M., Jr., "Summary of Available Information on Reynolds Analogy for Zero-Pressure-Gradient, Compressible, Turbulent-Boundary-Layer Flow," TN D-5560, Jan. 1970, NASA.

[26] Schlichting, H., *Boundary Layer Theory*, Pergamon Press, New York, 1955, p. 423.